Math 2130 - Homework # 3

Directional Derivative and Gradient

Part 1 - Basic computations

1. Find the rate of change of $f(x, y) = 5xy^2 - 4x^3y$ at $P = (1, 2)$ in the direction of $\vec{u} = \left\langle \frac{5}{13}, \frac{12}{13} \right\rangle$.
2. Find the directional derivative of $f(x, y) = \ln(x^2 + y^2)$ at $(1, 2)$ in the direction of $\vec{v} = \langle -1, 2 \rangle$.
3. Find the directional derivative of $g(x, y) = \sqrt{xy}$ at $P = (2, 8)$ in the direction of $Q = (5, 4)$.
4. Find the directional derivative of $h(x, y) = x^2 + y^2$ at $P = (2, 1)$ in the direction of $Q = (0, 0)$.

Part 2 -More in depth exploration

5. Let $f(x, y) = 2x + y$ and $P = (1, 0)$.
   (a) Draw the level curves of f when $k = -2, 0, 2$. What level curve does P lie on?
   (b) Find the direction of the maximal rate of increase of f at $P = (1, 0)$. What is the maximal rate of increase at P?
   (c) On your level curves picture add the vector $\nabla f(1, 0)$ that you found in part (b) with it's starting point put at $P = (1, 0)$. How does the vector $\nabla f(1, 0)$ relate to the level curve $k = 2$ that it starts at?
6. Let $f(x, y) = x^2 + y^2$ and $P = (0, 2)$.
   (a) Draw the level curves of f when $k = 1, 4, 9$. What level curve does P lie on?

(b) Find the direction of the maximal rate of increase of f at P. What is the maximal rate of increase at P?

(c) On your level curves picture add the vector $\nabla f(0,2)$ that you found in part (b) with it's starting point put at $P = (0,2)$. How does the vector $\nabla f(0,2)$ relate to the level curve $x^2 + y^2 = 4$ that it starts at?

(d) Find the directions $\vec{u}$ where $D_{\vec{u}} f(0,2) = 0$. Only find the unit vectors $\vec{u}$ that solve the equation.

(e) On your level curves picture add the vectors that you found in part (d). Have them start at $P = (0,2)$. How do they relate to the level curve $x^2 + y^2 = 4$ that goes through P ?
